Теоретическое описание стекольного перехода во фрустрированной XY модели

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Теоретическое описание стекольного перехода во фрустрированной XY модели

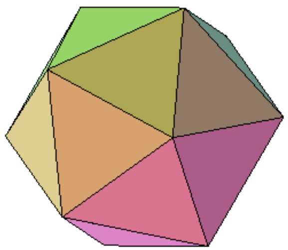
(в вырожденных SO(2) (U(1)) системах)

Conditions for glass transition:

- Ordering process (phase transition)
- Frustration

Frustration in amorphous matter

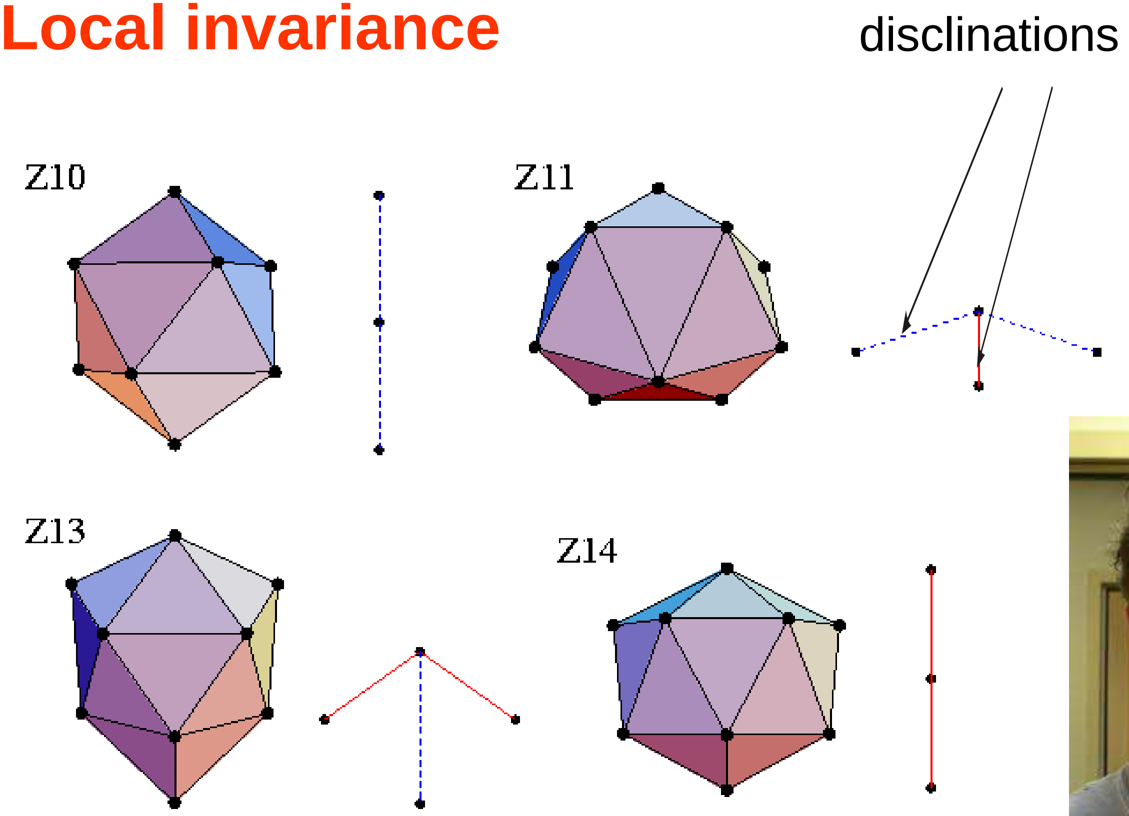
Geometrical frustration



- icosahedra

Local invariance

David R. Nelson

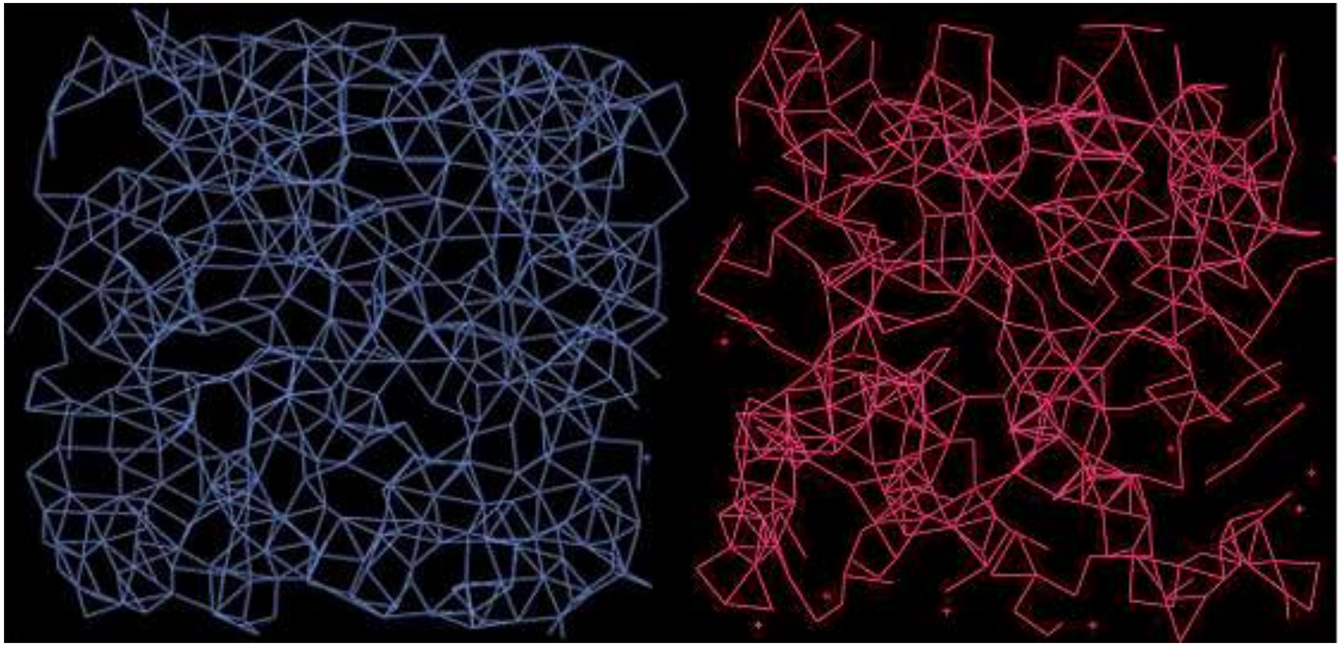


Gilles Tarjus

Z - coordination number

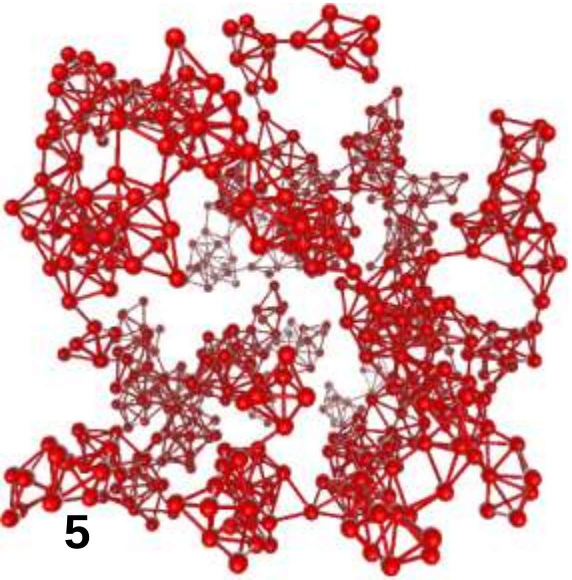
Disclination net of model of liquid Ni

axes 4-th order axes 6-th order



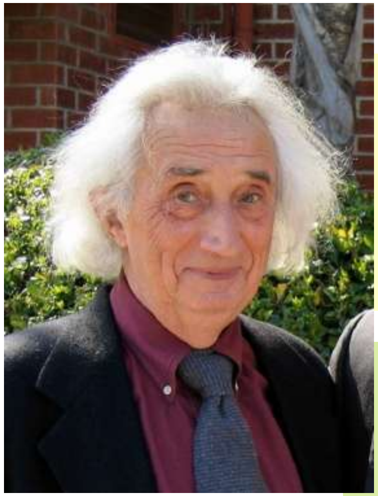
LAM 12, Metz 2004

1773 K



Poly-tetrahedral cluster

Gauge model of Glass



I. E. Dzyaloshinskii

LE JOURNAL DE PHYSIQUE TOME 39, JUIN 1978, PAGE 693

ON THE CONCEPT OF LOCAL INVARIANCE IN THE THEORY OF SPIN GLASSES

I. E. DZYALOSHINSKII and G. E. VOLOVIK

G. E. Volovik

Topological phase transition in the *XY* model of a spin glass

I. E. Dzyaloshinskiĭ and S. P. Obukhov

L. D. Landau Institute of Theoretical Physics, Academy of Sciences, USSR

(Submitted 13 April 1982)
Zh. Eksp. Teor. Fiz. **83**, 813–832 (August 1982)

PHYSICAL REVIEW B VOLUME 18, NUMBER 9

Gauge models for spin-glasses

J. A. Hertz

J. A. Hertz

J. Physique **43** (1982) 293-306

Line defects and tunnelling modes in glasses

N. Rivier and D. M. Duffy

Revista Brasileira de Física, Vol. 15, nº 4, 1985

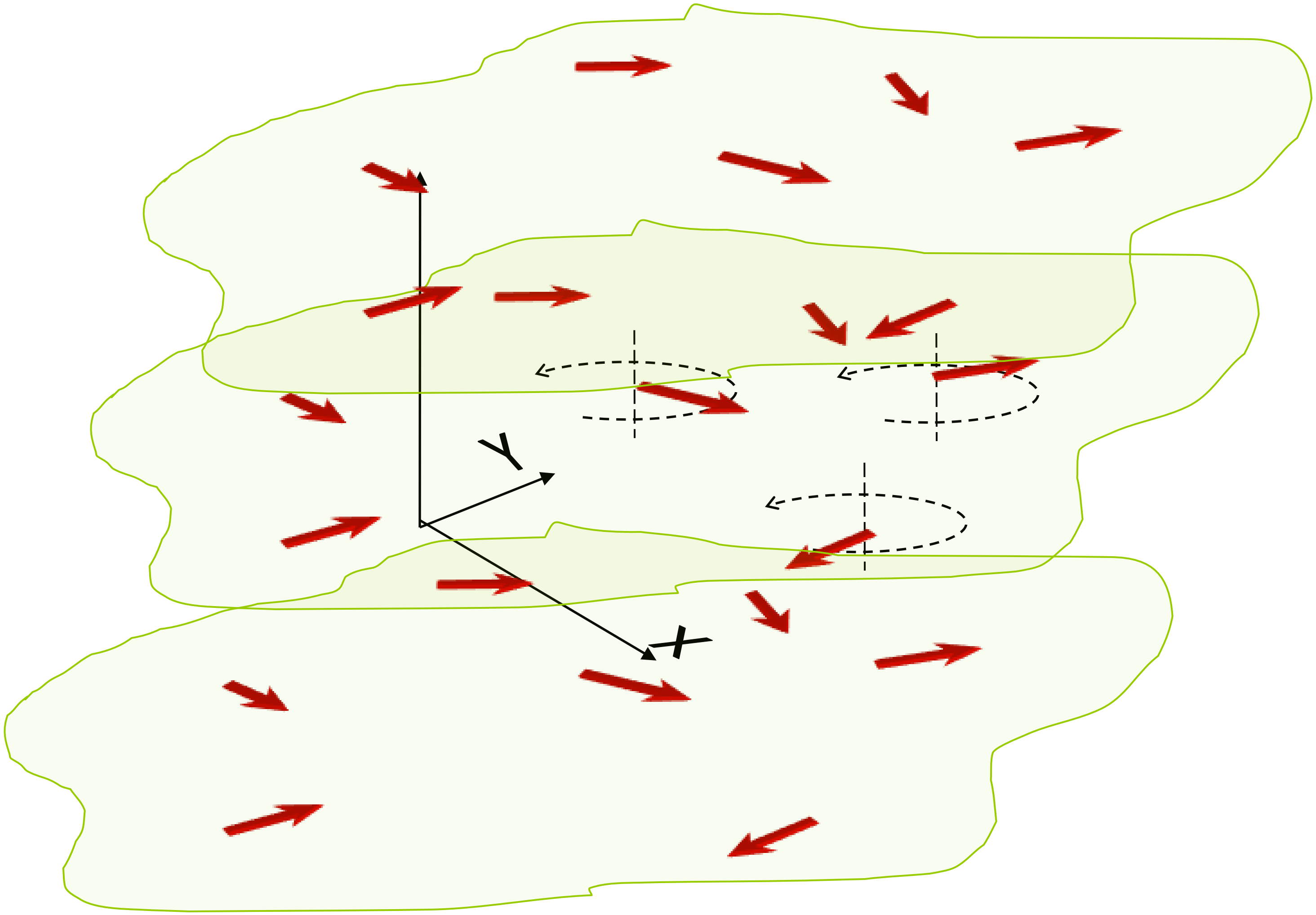
Theory of Glass*

N. RIVIER

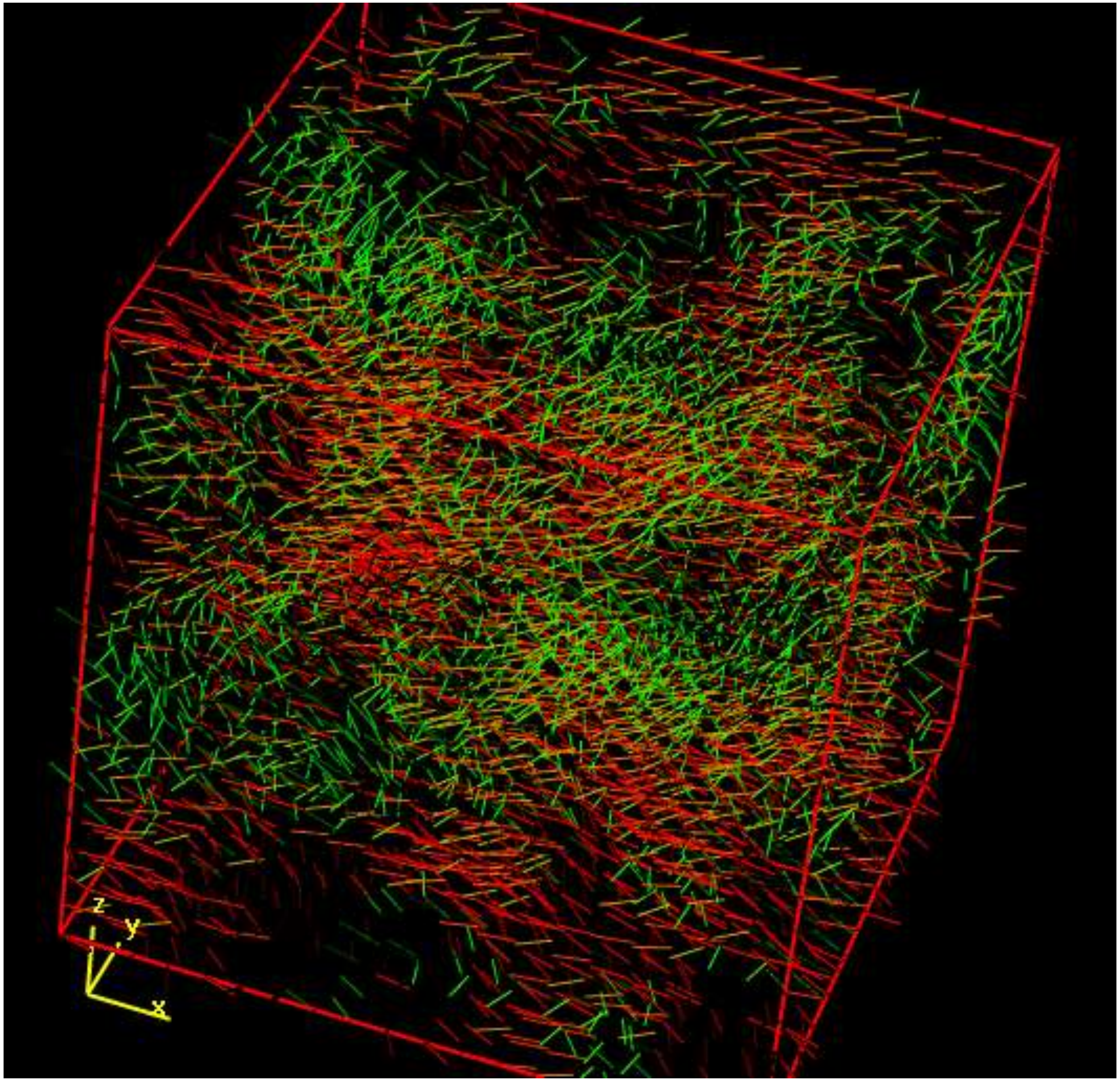
N. Rivier

3D XY-model

The simplest example of degenerate system



3D XY-model



XY-model

Continuous formulation

J. Villain, J. Phys. C **10**, 1717 and 4793 (1977); **11**, 745 (1978).

$$\mathcal{L} = |\partial_\nu \Psi|^2 + m|\Psi|^2 - \frac{b}{2}|\Psi|^4,$$

order parameter field
(vector in XY plane)

$$\Psi \to \Psi e^{i\phi}$$

potential energy - $U(\Psi) = -m|\Psi|^2 + \frac{b}{2}|\Psi|^4$

$$m = \alpha(T_c - T)\ (\alpha > 0)$$

temperature of symmetry breaking

temperature of the system

The system undergoes the symmetry breaking in T_c.

XY-model

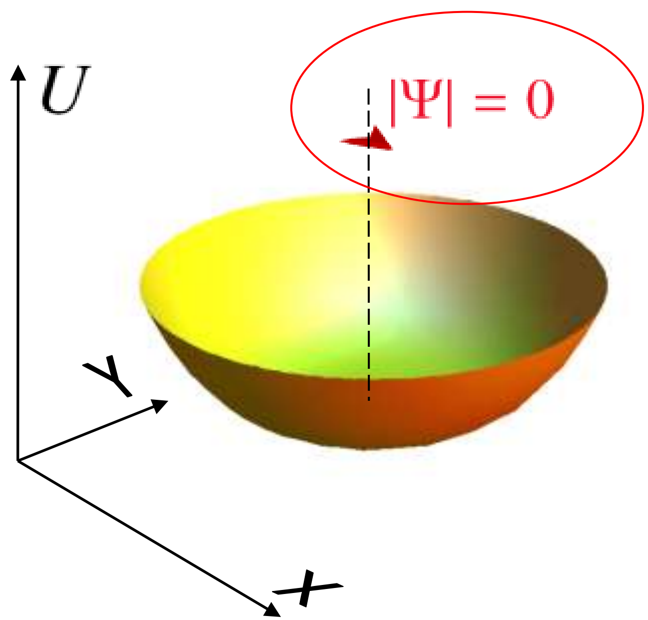
Spontaneous symmetry broken

$$U(\Psi) = -m|\Psi|^2 + \frac{b}{2}|\Psi|^4$$

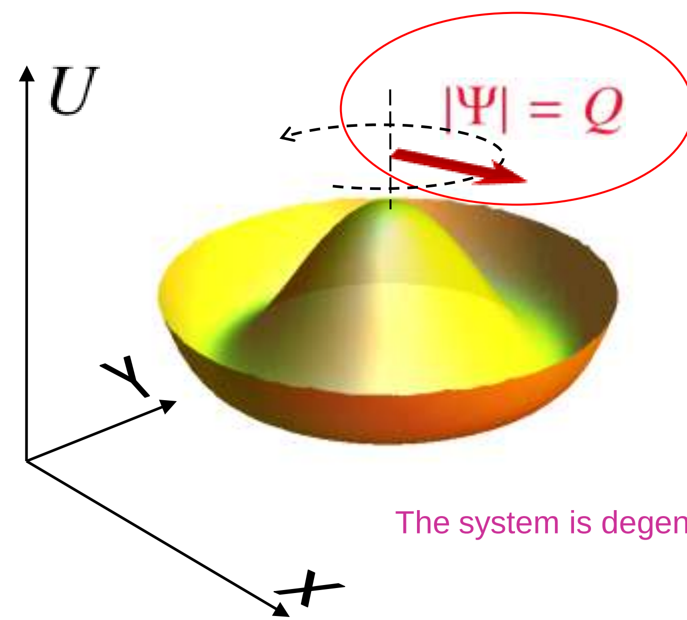
$$\Psi \rightarrow \Psi e^{i\phi}$$

local ordering

$$|\Psi|^2 = Q^2 = \alpha(T_c - T)/b$$



$m < 0$ $(T > T_c)$



The system is degenerated

$m > 0$ $(T < T_c)$

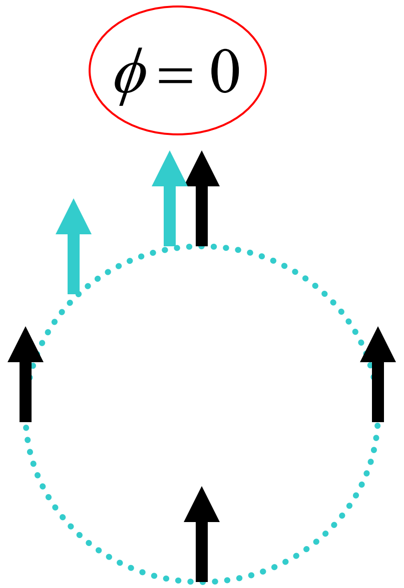
But it is not phase transition yet!

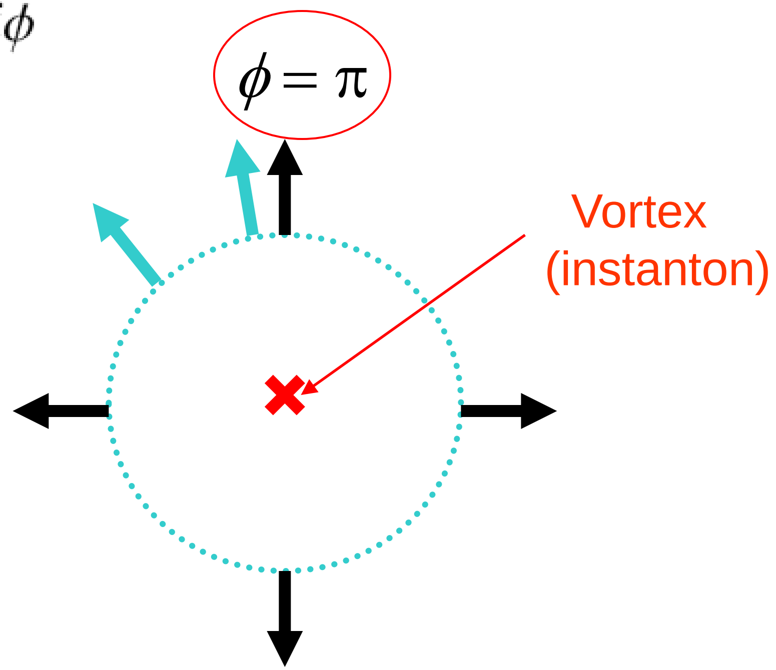
XY-model

The appearance of the non zero magnitude of the local order parameter does not mean the appearance of a long range order.

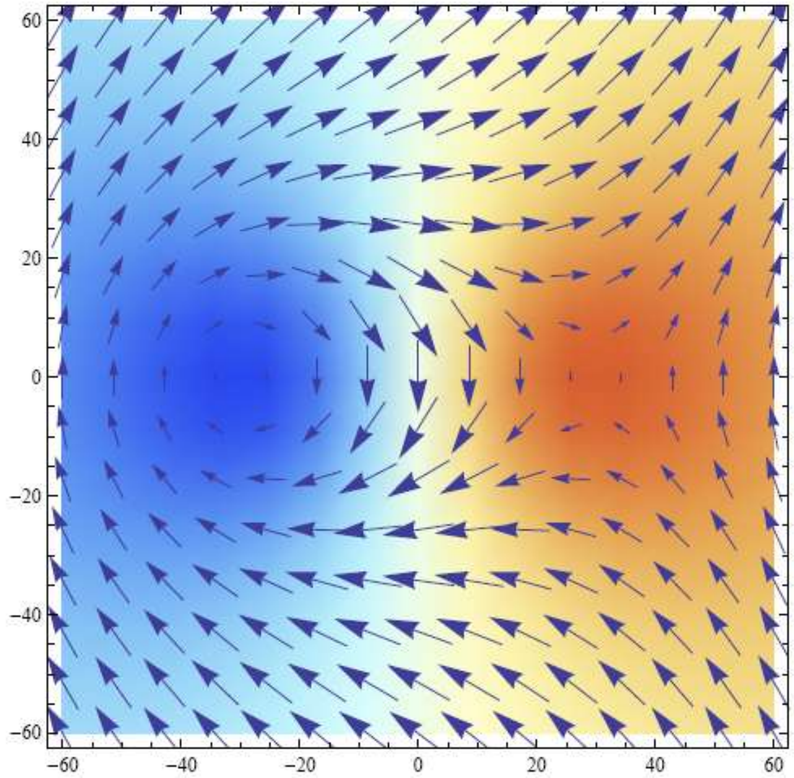
The reason is the presence of instanton solutions (disclinations or vortices).

$$\Psi \rightarrow \Psi e^{i\phi}$$





XY-model



- A vortex (disclination) is a topological stable spin-formation, which lowers the correlation of the system.
- There are two kinds of vortices (disclinations): clockwise and counterclockwise which occur as pairs (dipole).
- Because vortices (disclinations) are stable they can only disappear when a pair comes together and annihilates.

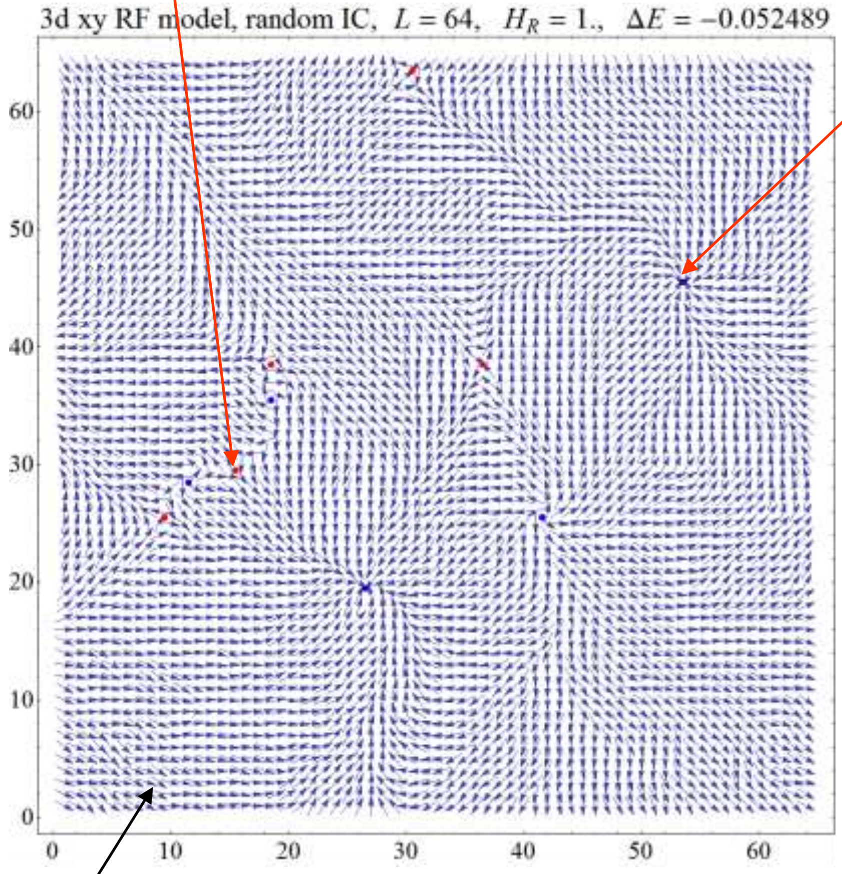
3D XY-model

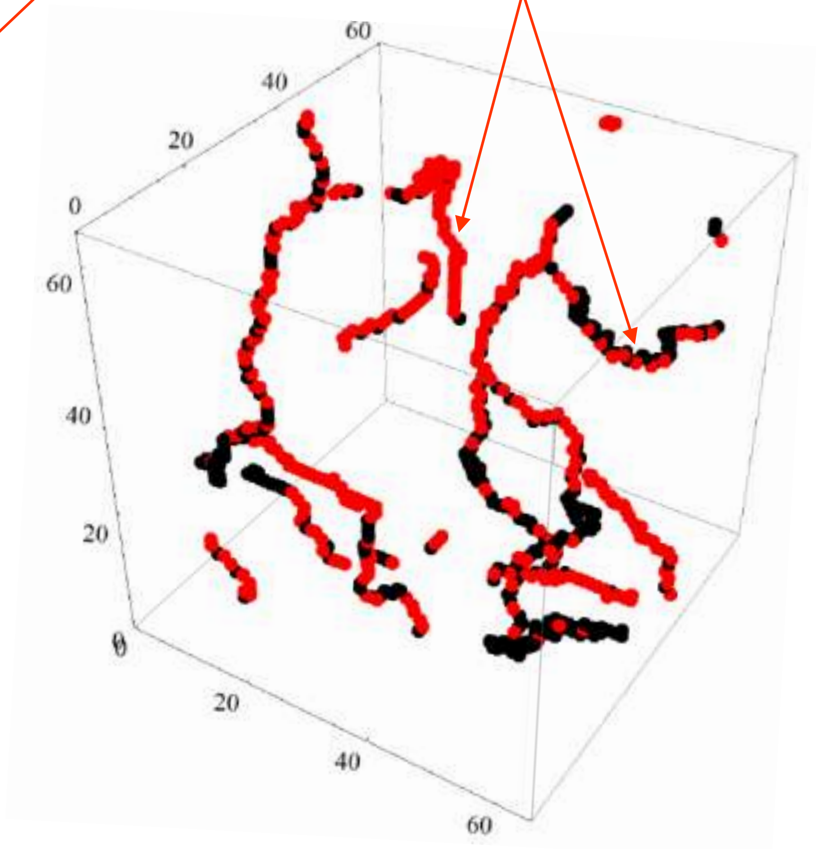
«Фрустрационныйе линии» I. E. Dzyaloshinskii and S. P. Obukhov, Sov. Phys. JETP 1982

vortex

vortex

vortex lines (disclinations)





cross-section

D.A.Garanin, E.M.Chudnovsky, T.Proctor Phys.Rev. B88 (2013) 22, 224418

XY-model

In order to take the disclinations into account in the model the derivative is changed by the covariant derivative with gage field A. The disclination is source of the gage field.

$$\mathcal{L} = |\partial_\nu \Psi|^2 + m|\Psi|^2 - \frac{b}{2}|\Psi|^4,$$

$$\mathcal{L} = -\frac{F_{\mu\nu}^2}{8\pi} + |(\partial_\nu - igA_\nu)\,\Psi|^2 + m|\Psi|^2 - \frac{b}{2}|\Psi|^4$$

covariant derivative

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\Psi \rightarrow \Psi e^{i\phi}$$

The system is invariant under rotation

XY-model

We consider static case

$$\partial_0 A_\nu = 0, \partial_0 \Psi = 0,$$

in the Coulomb gauge

$$A_\nu = (0, i\mathbf{A}), \nabla \cdot \mathbf{A} = 0$$

fluctuation near the equilibrium value $\psi = \Psi - Q$

$$|\Psi|^2 = Q^2 = \alpha(T_c - T)/b$$

$$\mathcal{F} = \frac{\mathbf{B}^2}{8\pi} + |\nabla\psi|^2 + \frac{M^2}{8\pi}\mathbf{A}^2 + m|\psi|^2 + g^2\mathbf{A}^2|\psi|^2 + \ldots$$

where $\mathbf{B} = \nabla \times \mathbf{A}$, and $M^2 = 4\pi g^2 \alpha(T - T_c)/b$

XY-model

We consider static case

$$\partial_0 A_\nu = 0, \; \partial_0 \Psi = 0,$$

in the Coulomb gauge

$$A_\nu = (0, \; i\mathbf{A}), \; \nabla \cdot \mathbf{A} = 0$$

fluctuation near the equilibrium value

$$\langle \psi(\mathbf{p})\psi(-\mathbf{p}) \rangle = G(\mathbf{p}) \qquad \langle A_\mu(\mathbf{p}) A_\nu(-\mathbf{p}) \rangle = \Delta_{\mu\nu}(\mathbf{p})$$

$$\psi = \Psi - Q, \qquad |\Psi|^2 = Q^2 = \alpha(T_c - T)/b$$

$$\mathcal{F} = \frac{\mathbf{B}^2}{8\pi} \ldots M^2 \ldots \psi|^2 + \ldots$$

$$G(\mathbf{p}) = \frac{\beta^{-1}}{\mathbf{p}^2 + m}, \qquad \Delta_{\mu\nu}(\mathbf{p}) = \frac{4\pi\beta^{-1}\delta_{\mu\nu}}{\mathbf{p}^2 + M^2},$$

where $\mathbf{B} = \nabla \times \mathbf{A}$, and $M^2 = 4\pi q^2 \alpha(T - T_c)/b$

Taking into account of vortices

averaging over all configurations of the disclinations:

$$Z=\sum_{N=1}^{\infty}\frac{\lambda^N}{N!}\sum_{\sqrt{\beta a^{D-2}}J_n=\pm 1}\int \mathcal{D}\mathbf{A}e^{-\beta\int \mathcal{F}d^Dr}\int\prod_{n=1}^{N}d\mathbf{r}_n\exp\left[i\beta a^{D-2}J_n|\mathbf{A}(\mathbf{r}_n)|\right].$$

Source of **A**

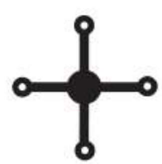
leads to

$$\mathcal{F}'=\frac{\mathbf{B}^2}{8\pi}+\frac{M^2}{8\pi}\mathbf{A}^2-2\lambda\beta^{-1}\cos(\sqrt{\beta a^{D-2}}|\mathbf{A}|),$$

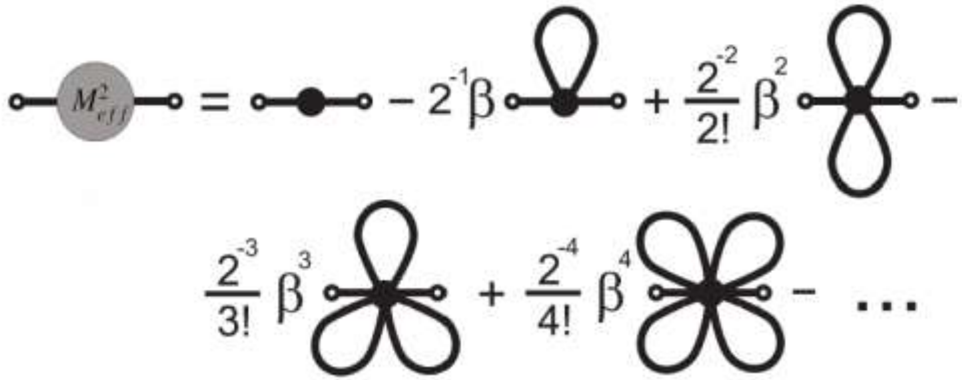
$$\text{where } \lambda=\lambda_0 e^{-\beta E_c}$$

a – is the radius of the disclination core

power series expansion of **cos**

cos $= 1-\beta\frac{1}{2!}$  $+\beta^2\frac{1}{4!}$  $-\beta^3\frac{1}{6!}$  $+\ldots$

2D XY-model



$$M^2_{eff} \approx M^2 + 16\pi\lambda \Big[1 - \frac{1}{2}\beta\Lambda(M^2) + \frac{3}{4!}\beta^2\Lambda^2(M^2) -$$

$$\frac{5\cdot 3}{6!}\beta^3\Lambda^3(M^2) + \ldots \Big] = M^2 + 16\pi\lambda \exp\left[-\frac{\beta}{2}\Lambda(M^2)\right],$$

$$\Lambda(M^2) \approx \beta^{-1}\int \frac{d^2p}{(2\pi)^2}\frac{4\pi}{\mathbf{p}^2+|M|^2} = \beta^{-1}\ln(1+a^{-2}/|M|^2).$$

We expect that $M^2 \ll a^{-2}$. Therefore

$$\boxed{M^2_{eff} \approx} M^2 + 16\pi\lambda \exp\left[-\frac{1}{2}\ln(a^{-2}/|M|^2)\right] = \boxed{M^2 + 16\pi\lambda\sqrt{|M|^2a^2}.}$$

3D XY-model

Члены разложения в степенной ряд с $n > 2D/(D-2)$

иррелевнтны (несущественны)!

$$\mathcal{F}' = \frac{\mathbf{B}^2}{8\pi} + \frac{M^2}{8\pi}\mathbf{A}^2 - 2\lambda\beta^{-1}\cos(\sqrt{\beta a^{D-2}}\,|\mathbf{A}|),$$

where $\lambda = \lambda_0 e^{-\beta E_c}$

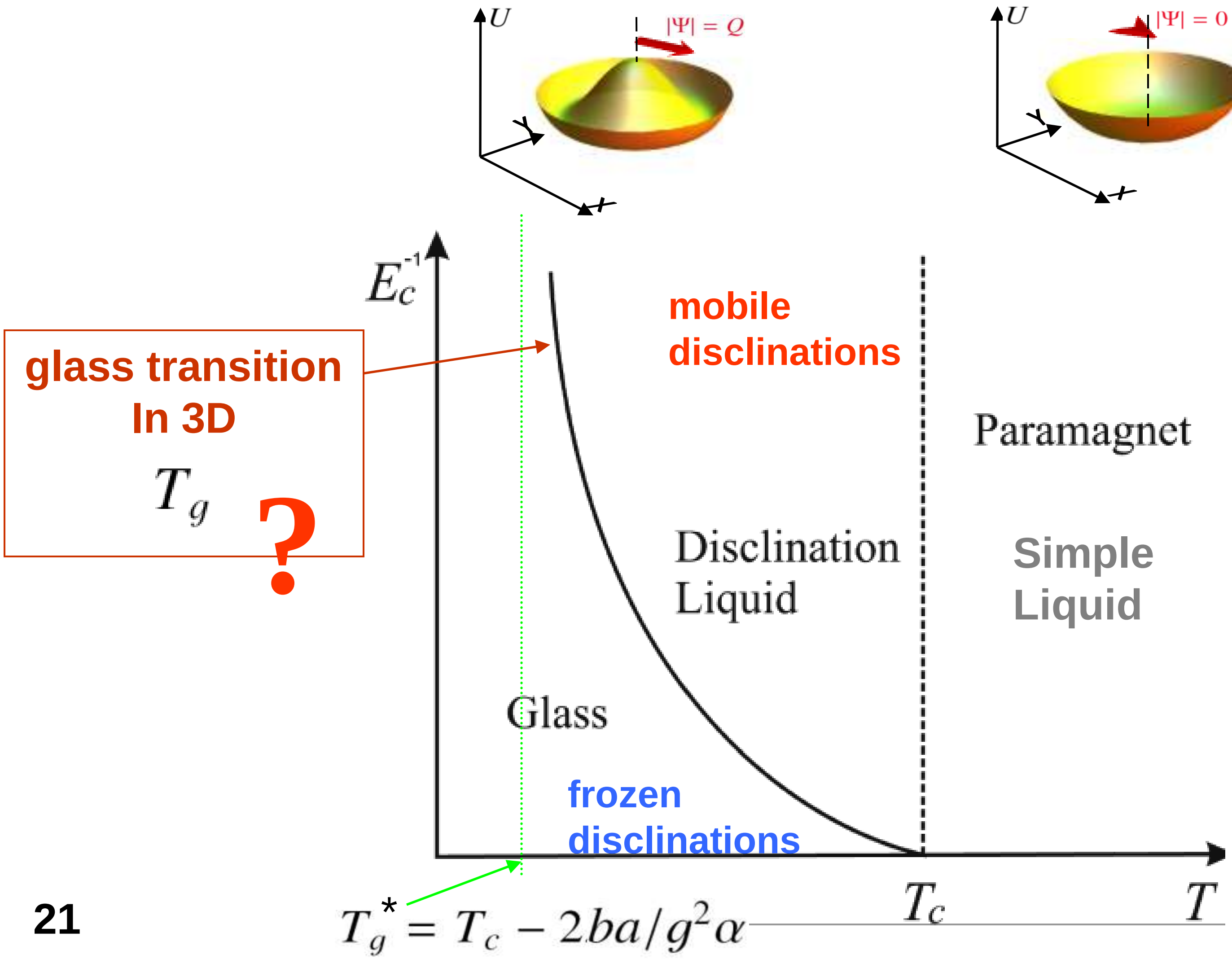
In 3D

$$\mathcal{F}' = \frac{\mathbf{B}^2}{8\pi} + \frac{M^2}{8\pi}\mathbf{A}^2 - \lambda\frac{\beta a^2}{4!}\mathbf{A}^4 + \lambda\frac{\beta^2 a^4}{6!}\mathbf{A}^6,$$

$$\boxed{M_{eff}^2 = 4\pi g^2\alpha(T - T_g)/b}$$

$$T_g = T_c - 2\lambda b/g^2\alpha$$

Phase diagram



Оценка корреляционного радиуса и времени релаксации

Correlation radius estimation

$$\langle \mathbf{J}(0)\mathbf{J}(\mathbf{r})\rangle \propto \exp[-r/r_c]$$

$$\langle \mathbf{J}(\mathbf{p})\mathbf{J}(-\mathbf{p})\rangle \propto (p^2 + r_c^{-2})^{-1}$$

$$\langle \mathbf{J}(\mathbf{p})\mathbf{J}(-\mathbf{p})\rangle = \int \mathcal{D}A\, \mathbf{J}(\mathbf{p})\mathbf{J}(-\mathbf{p}) \exp\left[-\beta \int \mathcal{F}' d\mathbf{r} - i\beta a^{D-2}\mathbf{J}(-\mathbf{p})\mathbf{A}(\mathbf{p}) + \right.$$
$$\left. i\beta a^{D-2}\mathbf{J}(\mathbf{p})\mathbf{A}(-\mathbf{p})\right] \propto \mathbf{J}(\mathbf{p})\mathbf{J}(-\mathbf{p}) \exp\left[\beta \mathbf{J}(\mathbf{p}) \frac{4\pi a^{2(D-2)-D}}{\mathbf{p}^2 + M_{eff}^2} \mathbf{J}(-\mathbf{p})\right],$$

$$\mathbf{J}(\mathbf{p})\mathbf{J}(-\mathbf{p})|_{p\to 0} = \int d\mathbf{r}\mathbf{J}^2(\mathbf{r}) = \beta^{-1} a^{2-D}$$

$$\boxed{r_c \propto \sqrt{\langle \mathbf{J}(\mathbf{p})\mathbf{J}(-\mathbf{p})\rangle}_{p\to 0} \propto \exp\left[2\pi a^{-2} M_{eff}^{-2}\right]}$$

Glass transition (3D)

Let us consider vortex correlation

$$\langle \mathbf{J}(\mathbf{p})\mathbf{J}(-\mathbf{p})\rangle = \int \mathcal{D}A\, \mathbf{J}(\mathbf{p})\mathbf{J}(-\mathbf{p}) \exp\left[-\beta \int \mathcal{F}' d\mathbf{r} - \beta a \mathbf{J}(-\mathbf{p})\mathbf{A}(\mathbf{p}) + \beta a \mathbf{J}(\mathbf{p})\mathbf{A}(-\mathbf{p})\right] \propto \mathbf{J}(\mathbf{p})\mathbf{J}(-\mathbf{p}) \exp\left[\beta \mathbf{J}(\mathbf{p}) \frac{4\pi a^{-1}}{\mathbf{p}^2 + M_{eff}^2} \mathbf{J}(-\mathbf{p})\right],$$

The correlation radius can be estimated:

$$r_c \propto \sqrt{\langle \mathbf{J}(\mathbf{p})\mathbf{J}(-\mathbf{p})\rangle_{p\to 0}} \propto \exp\left[2\pi a^{-2} M_{eff}^{-2}\right],$$

where

$$M_{eff}^2 = 4\pi g^2 \alpha (T_g - T)/b \qquad T_g = T_c - 2\lambda b a / g^2 \alpha$$

Glass transition (3D)

Let us consider vortex correlation

$$\langle \mathbf{J}(\mathbf{p})\mathbf{J}(-\mathbf{p})\rangle = \int \mathcal{D}A\, \mathbf{J}(\mathbf{p})\mathbf{J}(-\mathbf{p}) \exp\left[-\beta \int \mathcal{F}' d\mathbf{r} - \beta a \mathbf{J}(-\mathbf{p})\mathbf{A}(\mathbf{p}) + \beta a \mathbf{J}(\mathbf{p})\mathbf{A}(-\mathbf{p})\right] \propto \mathbf{J}(\mathbf{p})\mathbf{J}(-\mathbf{p}) \exp\left[\beta \mathbf{J}(\mathbf{p}) \frac{4\pi a^{-1}}{\mathbf{p}^2 + M_{eff}^2} \mathbf{J}(-\mathbf{p})\right],$$

The relaxation time is

$$\tau \propto \exp\left[\frac{b}{\alpha(ag)^2}\frac{1}{(T-T_g)}\right] = \exp\left[\frac{1}{2\lambda a^3}\frac{(T_c - T_g)}{(T - T_g)}\right]$$

Vogel-Fulcher-Tammann law

where

$$M_{eff}^2 = 4\pi g^2 \alpha (T_g - T)/b \qquad T_g = T_c - 2\lambda b a / g^2 \alpha$$

БКТ (2D)

The correlation radius can be estimated:

$$r_c \propto \sqrt{\langle \mathbf{J}(\mathbf{p})\mathbf{J}(-\mathbf{p})\rangle}_{p\to 0} \propto \exp\left[2\pi a^{-2} M_{eff}^{-2}\right],$$

The relaxation time is

$$\tau \propto \exp\left[\frac{1}{8\sqrt{2\pi(\lambda a^2)^3}}\sqrt{\frac{T_c - T^*}{T - T^*}}\right]$$

BKT law

where

$$M_{eff}^2 = 4\pi g^2 \alpha (T^* - T)/b$$

Critical dynamics (3D)

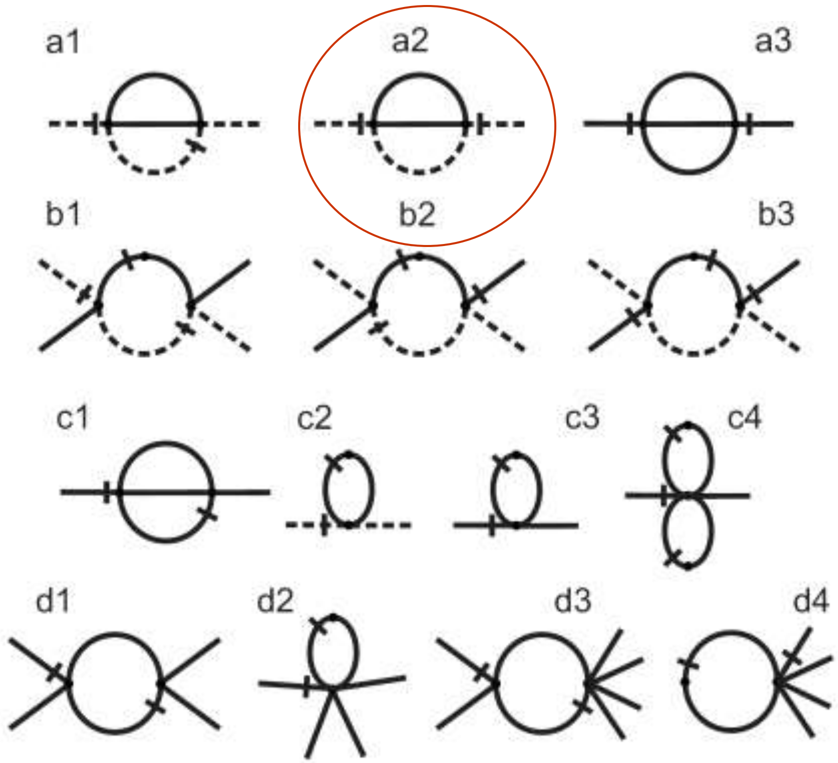
$$G^{R(A)}(\mathbf{p},\,\omega) = \frac{1}{\mathbf{p}^2 + m^2 \pm i\Gamma_\psi \omega}, \quad G^{K}(\mathbf{p},\,\omega) = \frac{2\Gamma_\psi T}{(\mathbf{p}^2 + m^2)^2 + \Gamma_\psi^2 \omega^2},$$

$$\langle \psi \bar{\psi} \rangle \qquad \langle \psi \psi \rangle$$

$$\Delta_{\mu\nu}^{R(A)}(\mathbf{p},\,\omega) = \frac{4\pi g_{\mu\nu}}{\mathbf{p}^2 + M_{eff}^2 \pm i\Gamma_A \omega}, \quad \Delta_{\mu\nu}^{K}(\mathbf{p},\,\omega) = \frac{2(4\pi)^2 \Gamma_A T g_{\mu\nu}}{(\mathbf{p}^2 + M_{eff}^2)^2 + \Gamma_A^2 \omega^2},$$

$$\langle A_\mu \bar{A}_\nu \rangle \qquad \langle A_\mu A_\nu \rangle$$

We can use the Keldysh technique to analyze the critical dynamics of the system.



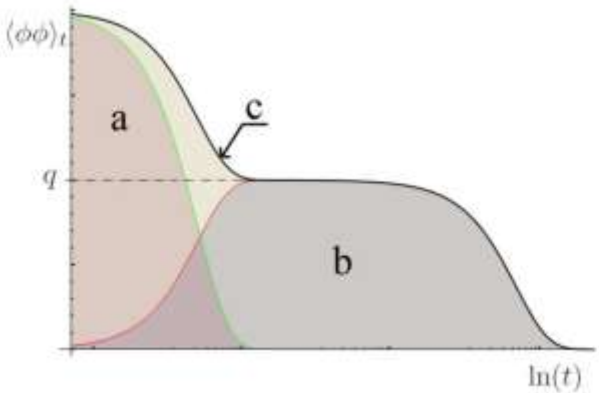
Critical dynamics (3D)

M. G. Vasin, Phys. Rev. B **74,** 214116 (2006);

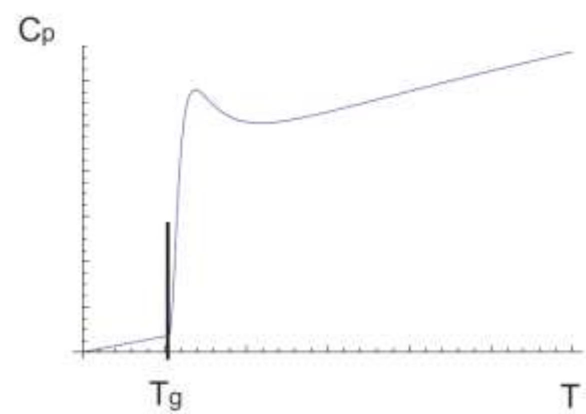
M.G. Vasin, N.M. Shchelkachev, and V.M. Vinokur, Theoretical and Mathematical Physics, 163(1): 537–548 (2010);

M. G. Vasin, J. Stat. Mech., P05009 (2011);

Plateau on the time dependence of the pair correlation function



Temperature dependence of heat capacity



The temperature dependence of the relaxation time satisfies to the Vogel-Fulcher-Tammann law

$$\tau \propto \exp\left[\frac{b}{\alpha(ag)^2}\frac{1}{(T-T_g)}\right] = \exp\left[\frac{1}{2\lambda a^3}\frac{(T_c-T_g)}{(T-T_g)}\right]$$

Выводы:

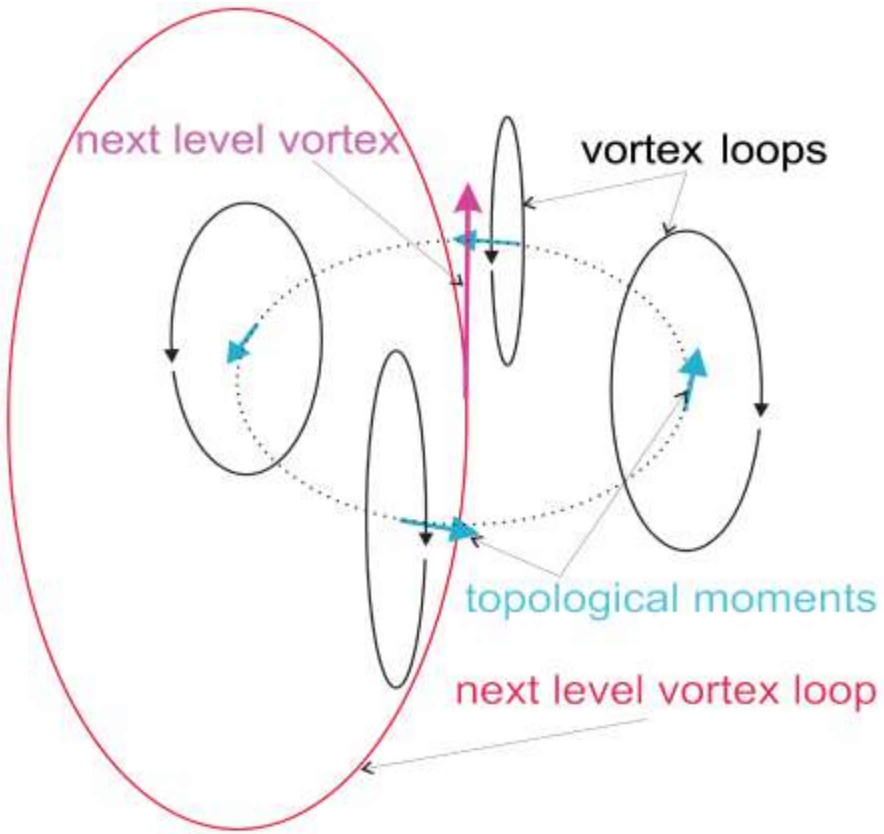
Переход Березинского-Костерлица-Таулеса и стекольный переход в XY-модели по своей природе есть аналогичный переход в 2D и 3D системах соответственно.

Он представляет собой топологический (геометрический) фазовый переход между состояниями с подвижными и неподвижными вихрями (дисклинациями).

В 2D системе тепловые флуктуации сильнее разрушают порядок, что приводит к более слабой расходимости корреляционного радиуса и характерного времени релаксации при приближении температуры к точке перехода.

Спасибо за внимание!

Critical dynamics (3D) [scaling]



The linear topological defects form the loops which have topological moments also forming a XY-system but at greater scales and so on.
This hierarchy of topological moments of the vortex loops allows us to use the scaling hypothesis for description of the critical properties of disordered systems.

Non-equilibrium quasi-long-range order of driven random field O(N) model

Taiki Haga
*Department of Physics, Kyoto University, Kyoto 606-8502, Japan**
(Dated: October 20, 2015)

We investigate three-dimensional O(N) spin models driven with a uniform velocity over a random field. Within a spin-wave approximation, it is shown that in the strong driving regime the model with $N = 2$ exhibits a quasi-long-range order in which the spatial correlation function decays in a power-law form. Furthermore, for the cases that $N = 2$ and 3, we numerically demonstrate a non-equilibrium phase transition between the quasi-long-range order phase and the disordered phase, which turns out to resemble the Kosterlitz-Thouless transition in the two-dimensional pure XY model in equilibrium.

The disclinations in the poly-tetrahedron model of simple liquid